Trial and Improvement

Solving Equations using Algebra

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We have seen how some equations can be solved using algebra.

$$2x - 8 = 0$$

$$2x = 8$$

$$x = \frac{8}{2}$$

$$x = 4$$

However...

Some equations are much harder to solve using algebra, such as:

$$x^3 - x = 12$$

or $$x^2 + 7x = 12$$

or $$x^3 - 4x = 7$$

Trial and Improvement

We can use Trial and Improvement to solve complicated equations. Although we cannot find the exact solution, we can find it to a certain precision.

Example 1:

Solve, correct to 1 decimal place

$$x^3 - 4x = 7$$

Step 1: Try a solution, eg $x = 1$

$$x^3 - 4x = (1)^3 - 4(1)$$
$$= 1 - 4$$
$$= -3$$

Step 2: Is our result too low or too high?

For $x = 1$, we found -3

This is too low, so try $x = 2$

Step 3: Try our new solution, $x = 2$

$$\begin{aligned} x^3 - 4x &= (2)^3 - 4(2) \\ &= 8 - 8 \\ &= 0 \end{aligned}$$

Again, this is too low. Try $x = 3$

Step 4: Try $x = 3$

$$\begin{aligned} x^3 - 4x &= (3)^3 - 4(3) \\ &= 27 - 12 \\ &= 15 \end{aligned}$$

Too high! Try $x = 2.5$

Step 5: Try $x = 2.5$

$$\begin{aligned} x^3 - 4x &= (2.5)^3 - 4(2.5) \\ &= 15.625 - 10 \\ &= 5.625 \end{aligned}$$

Too low! Try $x = 2.6$

Step 6: Try $x = 2.6$

$$\begin{aligned} x^3 - 4x &= (2.6)^3 - 4(2.6) \\ &= 17.576 - 10.4 \\ &= 7.176 \end{aligned}$$

Slightly too high!

But we've found our solution...

Step 7: We found

$$\text{for } x = 2.5 \quad \Rightarrow \quad 5.625$$
$$x = 2.6 \quad \Rightarrow \quad 7.176$$

We are trying to get 7 as a result, so x must be between 2.5 and 2.6

But we are trying to find it to 1dp, So we can use the closest result:

$$\underline{x = 2.6}$$

Example 2:

Solve, correct to 1 decimal place

$$x^2 + 2x = 5$$

Step 1:

Example 2:

Solve, correct to 1 decimal place

$$x^2 + 2x = 5$$

Step 1: Try $x = 1$

Example 2:

Solve, correct to 1 decimal place

$$x^2 + 2x = 5$$

Step 1: Try $x = 1$

$$\begin{aligned} x^2 + 2x &= (1)^2 + 2(1) \\ &= 1 + 2 \\ &= 3 \end{aligned}$$

Step 2:

Step 2: Try $x = 2$

Step 2: Try $x = 2$

$$\begin{aligned} x^2 + 2x &= (2)^2 + 2(2) \\ &= 4 + 4 \\ &= 8 \end{aligned}$$

Step 3:

Step 2: Try $x = 2$

$$\begin{aligned} x^2 + 2x &= (2)^2 + 2(2) \\ &= 4 + 4 \\ &= 8 \end{aligned}$$

Step 3: Try $x = 1.5$

Step 2: Try $x = 2$

$$\begin{aligned} x^2 + 2x &= (2)^2 + 2(2) \\ &= 4 + 4 \\ &= 8 \end{aligned}$$

Step 3: Try $x = 1.5$

$$\begin{aligned} x^2 + 2x &= (1.5)^2 + 2(1.5) \\ &= 2.25 + 3 \\ &= 5.25 \end{aligned}$$

Step 4:

Step 4: Try $x = 1.4$

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$$\begin{aligned} x^2 + 2x &= (1.4)^2 + 2(1.4) \\ &= 1.96 + 2.8 \\ &= 4.76 \end{aligned}$$

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Step 5: $x = 1.4 \quad \Rightarrow \quad 4.76$

$x = 1.5 \quad \Rightarrow \quad 5.25$

Step 4: Try $x = 1.4$

$$\begin{aligned} x^2 + 2x &= (1.4)^2 + 2(1.4) \\ &= 1.96 + 2.8 \\ &= 4.76 \end{aligned}$$

Step 5: $x = 1.4 \quad \Rightarrow \quad 4.76$

$x = 1.5 \quad \Rightarrow \quad 5.25$

So $x = 1.4$ (to 1 decimal place)